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ABSTRACT

This paper examines to show how an investigative approach to mathematics teaching can honor its two important dimensions--a thinking and reasoning side to go with the skill development. The paper addresses the question of why a problem solving approach has failed, makes the case for investigations as an overarching organizer, and discusses the make-up of an idealized classroom. (MM)

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Investigations as a Central Focus for a Mathematics Curriculum

by

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INVESTIGATIONS AS A CENTRAL FOCUS FOR A MATHEMATICS CURRICULUM

Charles Lovitt

It seems like forever we have been trying to say school mathematics has two equally important major dimensions — a thinking and reasoning side to go alongside the skill development .

The rise of the term Investigative thinking or Open-ended Investigations is the latest in what is beginning to be a long line of terms which attempt to capture and describe this thinking, reasoning side of our subject. None have completely succeeded.

Yet this time, I think there is a real chance for significant progress. Investigations offers something which the Problem Solving thrust did not. The Investigative Approach can provide the coherent 'big picture' showing how skills and problem solving can be balanced in harmony within a productive curriculum.

So this paper is an attempt to build an argument as to how this might happen. I would like to cover the following points each of which I hope is worthy of discussion over the term of this virtual conference.

1. Why has problem solving failed?
2. Investigations can provide an overarching organiser – a unifying set of criteria
3. An idealised classroom

1. Why has problem solving 'failed'?

I do not propose to spend much time on this, preferring to promote the potential benefits of Investigations, but there are important learnings from reflecting on the problem solving 'movement' from 1980 to the present.

I claim that problem solving has 'failed' — it arrived with great flourish and publicity in 1980 in both the AMEP [Australian Mathematics Education

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Program] and the American NCTM [National Council of Teachers of Mathematics] policy documents. Both, in a set of principles and standards argued 'Problem Solving should be the central focus' of a healthy mathematics curriculum.

If you just came down from Mars into 100 maths classrooms at random — you would be struggling to find compelling evidence of this 'central focus'. Why not?

I do accept that the problem solving push has contributed much to the vitality of many classrooms and significantly influenced the thinking of many teachers. But is has not become the 'central' theme it was supposed to be.

Two major reasons I believe are:

1. Lack of clear and widely accepted criteria. All sorts of things, some diametrically opposite to each other are all dressed up as problem solving. The word has become so blurred that we have no common shared agreement on what it means.

2. Another reason is the unfortunate perception that one aspect of the problem solving picture is delivered through games and puzzles and therefore is relegated to the periphery or margins of mathematics. 'I do these really interesting things on Friday afternoons … ' say many teachers to me. I am not sure if they are conscious that the act of doing so is to send a message to students that it is not really important — merely a bit of fun to be done after the 'real stuff'.

I hope some vigorous discussion may emanate from the above.
2. The Investigation Process can provide an overarching organisational framework for school mathematics

The investigative process can give us the central set of criteria that Problem Solving lacked, a possible unifying overview of our subject about which we could hope to get universal agreement . Its central theme is to present School Mathematics as close to what 'real' mathematicians do.

I take my inspiration for this argument from Professor Derek Holton. In an article titled What Mathematicians Do — And Why They Do It . (Best of

Set, ACER, Melb. 1994), he listed the process by which mathematicians create knowledge and solve problems. Paraphrasing this gives the following list of Stages.

1. Find an interesting (meaningful/worthwhile) problem.
2. Informally explore, unstructured 'play' which generates data.
3. From patterns in the data, create hypotheses, conjectures, theories.
4. Invoke problem solving strategies to prove or disprove any theories.
5. Apply any basic skills I know as part of this proof process.
6. Extend and generalise the problem – what else can I learn from it?
7. Publish (or perish).
8. Go back to step 1.

Derek Holton argues that this is how he spends his time as a professional mathematician. Firstly he finds an interesting problem. [Importantly if he fails Step 1, he does not go on - think of the ramifications of this for schooling]. Then he plays with, teases at the problem in an unstructured, informal, intuitive manner. This yields some information (data) out of which theories or conjectures form.

Now comes I think the important and unifying bits for schools – the next stage is to invoke problem solving strategies to help unlock the logic or structure of the problem. At this moment in the process you dip into your acquired tool-bag of problem solving strategies. ['I think I've seen a similar problem'... 'I could break this into more manageable parts'... 'I'll try some simpler or extreme cases'...] The greater the acquired set of strategies, the greater the probability of successfully unlocking the problem.

And Stage 5 — in the middle of this activity, the mathematician also calls up the acquired background set of basic skills — algorithms, graphing techniques, algebraic modelling, solution methods for equations, etc. The greater this background set of skills, the greater the probability of making progress in the problem.

Right there, at Stages 4 and 5, it seems to me the two interrelated themes of school mathematics come together, both in the service of a higher aim - as contributors to the investigative process.

Stage 6: 'What else can I learn? — how can I stretch the problem? — what if I change this variable? etc.'

Stage 7: I've gone as far as I am interested — I know there is more to discover, but I think its time for me to pull out — I'll write up what I've found so far as a possible journal article.

and then Stage 8 — go and find another interesting problem...

If this is approximately the process that 'real' mathematicians use, then should not school be some sort of a genuine imitation. So might these 8 stages become central organising criteria for curriculum?

Several supportive thoughts come to mind.

1. We have a greater chance of getting wide acceptance of this investigative structure as a central theme because it seems an honest reflection of 'real' mathematics.

2. The word 'Investigations' is not already overlaid with myriad linguistic interpretations that I think bedevilled Problem Solving from the start.

3. It shows how both strategies and skills work together in the service of a bigger challenge.

4. The stages provide easy to follow guidelines for both constructing and assessing curriculum. In presenting learning tasks to students, the stages provide a model or template for teacher action in the classroom. Secondly, when students present their findings, the stages provide a template for the communication. 'When you hand in your report', says the teacher, 'You need to show me how you...

* selected the task or challenge in the first place
* the results of the informal stage of 'playing' with the problem or challenge
* any observations, patterns, theories, conjectures you made
* the strategies you used to try to prove or disprove your theories
* the basic skills you used in this process e.g. algebra, graphing, calculus, spreadsheets etc.
* any extension possibilities you noticed or pursued...

... because these are the criteria by which I will assess your work!'

5. A diverse variety of learning challenges can fit such a structure from 'How can we control nuclear waste?' to 'How many different pairs of numbers share a LCM of 24?'

6. The structure works as well for Grade 2 as it does for Grade 12

7. The stages are a close reflection to the 'Scientific Method' widely used and accepted in science education

8. The structure seems consistent with prevailing learning theories

9. The structure seems consistent with many systemic Policy documents. For example, the 'Working Mathematically' strand of the National Mathematics Profile or its various State and Territory equivalents.

10. Maybe I'm on thin ground here, but more broadly than either maths or science, the stages seem to reflect the process by which a cultural creates and shares knowledge in the broadest and most fundamental of ways. Consider how we learn to throw a ball, or develop personal relationship skills or develop a new vaccine. Do not all of these arguably go through similar stages?

11. It's simple enough for everyone (importantly including students) to get their head around, yet not so simple as to be unfaithful to the deeper principles behind it.

3. An idealised classroom

Be it Grade 3 or Grade 11, I can imagine the following scenario.

1. When students enter 'their room', they know, because they have been trained accordingly, that the principles underpinning the course are the stages of the Investigative process. These are displayed prominently and permanently on the classroom walls and in the notebooks of students. These are the guiding and central principles behind their mathematics learning. Constantly referred to and promoted.

2. In the room there is a 'menu' of investigative offerings — suppose 100 investigations per major strand. For example, in Number, the menu might be:

N1 Steps
N2 The Year Puzzle
N3 Mirror Bounce
...
N100 Number Tiles

i.e. a huge variety of situational based rich open-ended challenges, all involving the theme of number as well as connections and links to other topics and subjects. All the investigations must pass the criteria or 'quality' control for inclusion in the menu in that they reflect most or all of the Investigative Stages.

A school may set itself the challenge of having 100 such Investigations in each of the 5 major content strands.

3. The students are assisted to navigate their way through the menu - that is, they make judicial selections of what investigations they may wish to pursue or indeed create new ones of interest to them or because it entails some event of current or personal interest.

4. Building in skills and strategies. Supposing the class has a cycle of 5 periods of Mathematics, the students know in advance that three of these will be devoted to building up their toolbox of skills. In this way the structure accommodates existing practice while at the same time showing students more directly the reason the toolbox is being developed. In a similar way, specific occasional lessons can be devoted to the toolbox of thinking strategies. Such as those that many schools call a strategy board. For example:

* act it out
* draw a diagram
* solve a simpler problem
* make a model
* work backwards
* test every possible case
* etc.

The teachers also have a collection of tasks, the purpose of which is to systematically teach the power and potential of each of the strategies.

Then for two of the five periods, the students are 'liberated' to be independent investigators, choosing from the Menu of offerings.

In my scenario, I have deliberately allocated the majority of time to the somewhat traditional acquisition of skills, though clearly this is flexible.

Conclusion

In our endless search for a rich, balanced and appropriate mathematics curriculum, open-ended investigative approaches may offer just the right structure.

I hope the above thoughts contribute to the healthy debate I am sure this virtual conference will generate.